



## Antiphased cyclotron-magnetoplasma mode in a quantum Hall system

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An antiphased magnetoplasma (MP) mode in a two-dimensional electron gas has been studied by means of inelastic light-scattering spectroscopy. Unlike the cophased MP mode, it is purely quantum excitation which has no classic plasma analog. It is found that zero-momentum degeneracy for the antiphased and cophased modes predicted by the first-order perturbation approach in terms of the electron-electron interaction is lifted. The zero-momentum energy gap is determined by a negative correlation shift of the antiphased mode. This shift, observed experimentally and calculated theoretically within the second-order perturbation approach, is proportional to the effective Rydberg constant in a semiconductor material.

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The unique symmetry properties of the quantum Hall (QH) electron liquid have stimulated progress in the study of strongly correlated electron systems in perpendicular magnetic field. In particular, it has been discovered that the simplest excitations of a two-dimensional electron gas (2DEG) are excitons consisting of an electron promoted from a filled Landau level (LL) and bound to an effective hole left in the “initial” LL.<sup>1–3</sup> Within the exciton paradigm, the physics of this many-particle quantum system is reduced to a *two-particle* problem. This can be solved in an asymptotically exact way where the parameter  $r_c = E_C / \hbar\omega_c$  is considered to be small. Here  $E_C = \alpha e^2 / \kappa l_B$  is the characteristic Coulomb energy,  $\omega_c$  is the cyclotron frequency, and the numerical coefficient  $\alpha < 1$  represents the averaged renormalization factor due to the finite thickness of the 2DEG in experimentally accessible systems. The excitation energy in this approach is the sum of two terms: (i) a single-electron gap (which is the Zeeman or cyclotron, or combined one) and (ii) a correlation shift induced by the electron-electron ( $e-e$ ) interaction. Kohn’s renowned theorem dictates that in a translationally invariant electron system, one of the excitons [magnetoplasma (MP) mode] has no correlation shift at  $q=0$ . This mode is described by the action of Kohn’s “raising” operator  $\hat{K}_s^\dagger = \sum_{np\sigma} \sqrt{n+1} c_{n+1,p,\sigma}^\dagger c_{n,p,\sigma}$  on the 2DEG ground state  $|0\rangle$ , where  $c_{n,p,\sigma}$  is the Fermi annihilation operator corresponding to the state  $(n,p)$  with the spin index  $\sigma = \uparrow, \downarrow$  ( $n$  is the LL number;  $p$  labels the inner LL number, if, e.g., the Landau gauge is chosen).<sup>4</sup> Yet, Kohn’s theorem *does not ban* the existence of another homogeneous MP mode that has a nonvanishing correlation shift. Precisely two MP modes should coexist at odd electron fillings  $\nu > 1$  when the numbers of fully filled spin sublevels differ by unit (see the illustration in Fig. 1). The symmetric mode is a cophased (CP) oscillation of spin-up and spin-down electrons and the antisymmetric one is an antiphased (AP) oscillation of two spin subsystems. When calculated to first order in terms of the parameter  $r_c$ , Kohn’s mode (the CP magnetoplasmon) has the energy<sup>3,5</sup>

$$E_s(q) = \hbar\omega_c + ve^2q/2\kappa + O(E_Cq^2l_B^2) \quad (1)$$

at small  $q$  ( $ql_B \ll 1$ ). The AP mode is a state orthogonal to  $\hat{K}_s^\dagger|0\rangle$ . It has the energy  $E_a(q) = \hbar\omega_c + O(E_Cq^2l_B^2)$  calculated to

first order in  $r_c$ .<sup>3</sup> Both Coulomb shifts  $\Delta_{s,a} = E_{s,a}(0) - \hbar\omega_c$  thus vanish if calculated up to  $\sim r_c$ . So, within this approximation, both MP modes turn out to be degenerate at  $q=0$ .

Kohn’s MP mode has been a prime subject for cyclotron resonance studies and the validity of Kohn’s theorem has been confirmed scores of times.<sup>6</sup> It is well established experimentally that homogeneous electromagnetic radiation incident on a translationally invariant electron system is unable to excite internal degrees of freedom associated with the Coulomb interaction, i.e.,  $\Delta_s \equiv 0$ . No similar experiments have been performed for the AP mode as it is not active in the absorption of electromagnetic radiation. Recent development of Raman-scattering spectroscopy to the point when it became sensitive to the cyclotron spin-flip and spin-density excitations<sup>7–10</sup> opened the opportunity to employ this spectroscopy in the investigation of the AP mode. Here, we report on a direct observation of the AP mode for a number of odd electron fillings and show that the theoretically predicted zero-momentum degeneracy for Kohn’s and AP modes is in fact lifted due to many-particle correlations. We also show that the *second-order corrections* to the excitation energies accurately reproduce the observed effect. The correlation shift for the AP mode is nonvanishing and negative at  $q=0$ .<sup>11</sup> In closing the introductory part it is worth noting: our study of the splitting between CP and AP modes which appears at

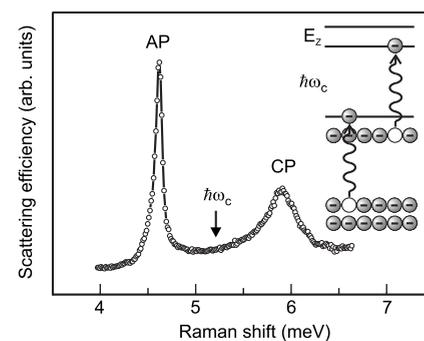


FIG. 1. ILS spectra of Kohn’s (CP) and antiphased (AP) magnetoplasma modes taken at  $\nu=3$ . The arrow indicates the cyclotron energy. The picture illustrates two single-electron transitions at odd filling factors that when coupled by the Coulomb interaction give rise to two magnetoplasma modes.

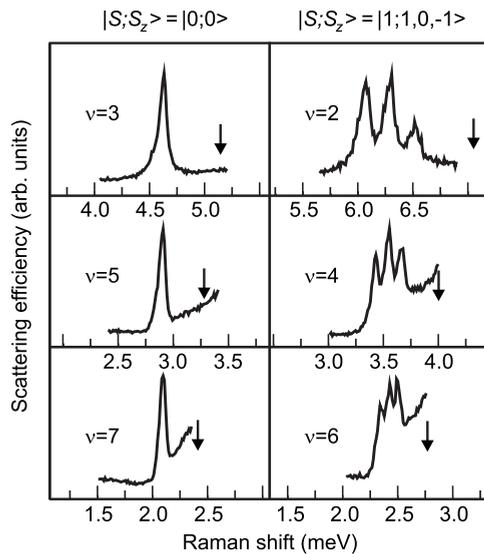


FIG. 2. ILS spectra of the AP magnetoplasma mode (left) and three Zeeman components of the cyclotron spin-flip mode (right) taken at odd (left) and even (right) filling factors. The arrows indicate the corresponding cyclotron energies.

second and higher order in electron-electron interactions should not be confused with the first-order splitting which is band-structure driven but influenced by interactions.<sup>12</sup>

Several high-quality heterostructures were studied. Each consisted of a narrow 18–20 nm GaAs/Al<sub>0.3</sub>Ga<sub>0.7</sub>As quantum well (QW) with an electron density of  $(1.2\text{--}2.4) \times 10^{11} \text{ cm}^{-2}$ . The mobilities were  $(3\text{--}5) \times 10^6 \text{ cm}^2/\text{Vs}$ —very high for such narrow QWs. The electron densities were tuned via the optodepletion effect and were measured by means of *in situ* photoluminescence. The experiment was performed at a temperature of 0.3 K. The QWs were set on a rotating sample holder in a cryostat with a 15 T magnet. The angle between the sample surface and the magnetic field was varied *in situ*. By continuously tuning the angle, we were able to increase the Zeeman energy while keeping the cyclotron energy fixed. This reduced thermal spin-flip excitations through the Zeeman gap. The inelastic light-scattering (ILS) spectra were obtained using a Ti:sapphire laser tunable above the fundamental band gap of the QW. The power density was below  $0.02 \text{ W/cm}^2$ . A two-fiber optical system was employed in the experiments.<sup>13</sup> One fiber transmitted the pumping laser beam to the sample; the second collected the scattered light and guided it out of the cryostat. The scattered light was dispersed by a Raman spectrograph and recorded with a charge-coupled device camera. Spectral resolution of the system was about 0.03 meV.

Narrow QWs were chosen to maximize energy gaps separating the size-quantized electron subbands. This mitigated the subband mixing induced by the tilted magnetic field. Yet, the mixing effect was important and we put it under close scrutiny. The influence of the tilted magnetic field on the cyclotron energy was studied for every QW by measuring the energies and dispersions for the MP and Bernstein modes.<sup>13</sup> Most accurately, this procedure was performed for the narrowest 18 nm QW where the nonlinearity was fairly small. Besides, it is exactly the 18 nm QW where the correlation

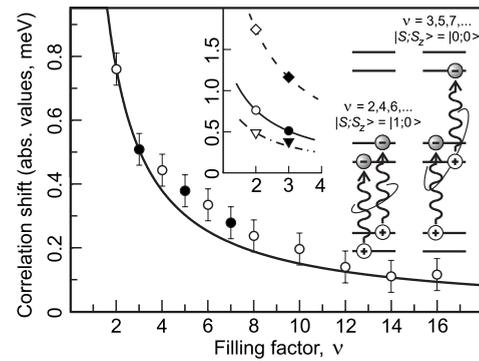


FIG. 3. Main picture: correlation shifts for the AP magnetoplasma mode (solid dots) and for the  $|S; S_z\rangle=|1; 0\rangle$  component of the cyclotron spin-flip mode (open dots). The solid line shows the  $2|\Delta_{\text{SF}}^{(\nu=2)}|/\nu$  dependence. In the inset, theoretical values for  $|\Delta_{\text{SF}}|$  and  $|\Delta_a|$  at  $\nu=2$  and  $\nu=3$  found for the self-consistently computed form factor  $F(q)$  (triangles) and for the strict 2D limit  $F(q)=1$  (diamonds). Corresponding functions  $2|\Delta_{\text{SF}}^{(\nu=2)}|/\nu$  are shown by dashed and dot-dashed lines. Circles and the solid line represent the experimental data. Illustration of single-electron transitions involved in the  $|S; S_z\rangle=|1; 0\rangle$  component of the cyclotron spin-flip triplet ( $\nu=2, 4, 6, \dots$ ) and in the AP mode ( $\nu=3, 5, 7, \dots$ ) is given on the right.

shift reaches its largest value, as it is affected by the QW width through the renormalization factor  $\alpha$ . Therefore, hereafter we will only address the 18 nm QW.

The ILS resonances for both CP and AP modes are shown in Fig. 1. They have quite different properties. Kohn's resonance is blueshifted from the cyclotron energy. Its small momenta dispersion is given by Eq. (1). Experimentally,  $q$  is defined by the orientation of pumping and collecting fibers relative to the sample surface. It is  $0.7 \times 10^5 \text{ cm}^{-1}$  for the spectra in Fig. 1. Kohn's resonance is well broadened because of linear  $q$  dispersion (1) and because the momentum is effectively integrated in the range of  $(q \sim 0.6\text{--}0.8) \times 10^5 \text{ cm}^{-1}$  due to the finite dimension of the fibers. On the contrary, the resonance for the AP mode is redshifted and does not broaden. In fact, we did not see any appreciable change in the AP mode energy upon varying the momentum transferred to the 2DEG via the ILS process. This experimental finding agrees with the first-order perturbation theory of Ref. 3, which predicts a negligible (compared to the experimental resolution) change in the AP mode energy at small  $q$ , defined by the light momentum. Variation in the AP shift in the accessible range of magnetic fields and electron densities is also within the experimental uncertainty. Since dimensional analysis of second-order Coulomb corrections to the energies of inter-LL excitations yields exactly an independence of the correlation shift on the magnetic field, we assume that the origin of the AP shift should be sought within the second-order perturbation theory.<sup>8,14</sup>

The redshift for the AP mode at odd  $\nu$  (QH *ferromagnets*) is filling factor dependent; it reduces at larger  $\nu$  (Fig. 2). Interestingly its value falls on the same  $1/\nu$  curve that describes the correlation shifts for the antisymmetric mode in another QH system, namely, that for the *cyclotron spin-flip mode in a spin-unpolarized 2DEG* at even  $\nu$  (Fig. 3). These two kinds of excitations differ by the total spin quantum

number:  $S=0$  for the AP mode which is a spinless magneto-plasmon and  $S=1$  for the cyclotron spin-flip mode. The latter splits into three Zeeman components with different spin projections along the magnetic field. As a consequence, in the experimental spectra of Fig. 2 a single ILS resonance corresponds to the AP mode, whereas the cyclotron spin-flip mode is represented by the Zeeman triplet. The  $e$ - $e$  correlation nature of redshift for the cyclotron spin-flip mode is confirmed theoretically in our previous publications<sup>8,14</sup> and here we employ a similar approach to calculate the AP shift at  $\nu=3$ .

Our technique is a variation in the standard perturbative technique,<sup>15</sup> although it has some special features. The first is the usage of the excitonic representation,<sup>14,16</sup> where the basis of exciton states is employed instead of degenerate single-electron LL states. Second, in the development of the perturbative approach, one is forced to use a nonorthogonal basis of two-exciton states. These are created by action of the interaction Hamiltonian on the single-exciton basis, when considering first-order corrections to the exciton states. The third feature lies in calculating the correlation shift counted from the ground-state energy and the latter also has to be taken into account up to the second-order corrections.

Because of the twofold degeneracy of the  $q=0$  MP states, we have to employ two single-exciton states as a bare basis set. As a result, we come to a  $2 \times 2$  secular equation. The bare states are  $|X_{\downarrow}\rangle = Q_{01}^{\dagger}|0\rangle$  and  $|X_{\uparrow}\rangle = Q_{12}^{\dagger}|0\rangle$ , where  $Q_{mk}^{\dagger} = Q_{mkq}^{\dagger}|_{q=0}$ , and  $Q_{mk}^{\dagger} = Q_{mkq}^{\dagger}|_{q=0}$ , and the exciton operators are defined, e.g., as<sup>14,16,17</sup>

$$Q_{mkq}^{\dagger} = \frac{1}{\sqrt{N_{\phi}}} \sum_p e^{-iq_x p} c_{k,p+q_y/2,\uparrow}^{\dagger} c_{m,p-q_y/2,\uparrow} \quad (2)$$

( $Q_{mkq}^{\dagger}$  differs by changing  $\uparrow$  to  $\downarrow$  in the right-hand side);  $q$  is

$$M_{\downarrow\downarrow} = -\frac{(e^2/\kappa l_B)^2}{4\hbar\omega_c} \sum_{\sigma_1, \sigma_2} \sum_{\substack{n_1, n_2, \mathbf{q} \\ m_1, m_2}} \sum_{\substack{n'_1, n'_2, \mathbf{q}' \\ m'_1, m'_2}} \left( \frac{\langle 0 | \left[ Q_{01}^{\dagger}, \hat{H}_{n'_1 n'_2 \mathbf{q}'}^{\sigma_1 \sigma_2} \right] \left[ \hat{H}_{n_1 n_2 \mathbf{q}}^{\sigma_1 \sigma_2}, Q_{01}^{\dagger} \right] | 0 \rangle}{n_1 + n_2 - m_1 - m_2} + N_{\phi}^{1/2} \frac{\langle 0 | \hat{H}_{n'_1 n'_2 \mathbf{q}'}^{\sigma_1 \sigma_2} \left[ Q_{00}^{\dagger} - Q_{11}^{\dagger}, \hat{H}_{n_1 n_2 \mathbf{q}}^{\sigma_1 \sigma_2} \right] | 0 \rangle}{n_1 + n_2 - m_1 - m_2} \right). \quad (5)$$

The LL number indexes  $n_i, n'_i$  and  $m_i, m'_i$  run from zero to infinity; however, only terms for which  $n_1 + n_2 - m_1 - m_2 = n'_1 + n'_2 - m'_1 - m'_2 \geq 1$  contribute to the total sum (5). (Other terms, being not subject to this condition, have zero numerators.) The expression for another diagonal matrix element  $M_{\uparrow\uparrow}$  differs from Eq. (5) by replacements  $Q_{01}^{\dagger} \rightarrow Q_{12}$ ,  $Q_{01}^{\dagger} \rightarrow Q_{12}^{\dagger}$ ,  $Q_{00}^{\dagger} \rightarrow Q_{11}$ , and  $Q_{11}^{\dagger} \rightarrow Q_{22}$ ; whereas the nondiagonal element  $M_{\downarrow\uparrow}$  differs from expression (5) by the absence of the second term in parentheses and the change from  $Q_{01}^{\dagger}$  to  $Q_{12}^{\dagger}$  in the first term. Correspondingly,  $M_{\uparrow\downarrow}$  is also obtained by omitting the second term and replacing  $Q_{01}^{\dagger}$  with  $Q_{12}$ . Analysis shows that  $M_{\downarrow\uparrow} \equiv M_{\uparrow\downarrow}$ , as it should be (both values are real).

Fortunately, the symmetry of the system and Kohn's theorem simplify the calculations a great deal. First, note that one

measured in units of  $1/l_B$  and  $N_{\phi}$  is the LL degeneracy number. The commutation rules of exciton operators define a special Lie algebra. Considering  $\hat{\mathcal{H}}_{\text{int}}$  as a part of the interaction Hamiltonian relevant to the calculation of the second-order energy corrections, we present it as a combination of two-exciton operators,

$$\hat{\mathcal{H}}_{\text{int}} = \frac{e^2}{2\kappa l_B} \sum_{\substack{n_1, n_2, \mathbf{q} \\ m_1, m_2}} \left( \hat{H}_{n_1 n_2 \mathbf{q}}^{\downarrow\downarrow} + 2\hat{H}_{n_1 n_2 \mathbf{q}}^{\downarrow\uparrow} + \hat{H}_{n_1 n_2 \mathbf{q}}^{\uparrow\uparrow} \right), \quad (3)$$

where  $\hat{H}_{n_1 n_2 \mathbf{q}}^{\uparrow\downarrow} = V(q) h_{m_1 n_1}(\mathbf{q}) h_{m_2 n_2}(-\mathbf{q}) Q_{m_1 n_1 \mathbf{q}}^{\dagger} Q_{m_2 n_2 - \mathbf{q}}^{\dagger}$ ,  $2\pi V(q)$  is the dimensionless two-dimensional (2D) Fourier component of the Coulomb potential,  $h_{mn}(\mathbf{q}) = (m!/n!)^{1/2} e^{-q^2/4} (q_-)^{n-m} L_m^{n-m}(q^2/2)$  [ $L_m^n$  is the Laguerre polynomial,  $q_{\pm} = \mp \frac{i}{\sqrt{2}}(q_x \pm iq_y)$ ]. Expressions for first and third operators in parentheses in Eq. (3) differ from the expression for  $\hat{H}_{\dots}^{\uparrow\downarrow}$  by replacement of  $Q^{\dagger}$  operators' indexes:  $m_2 n_2 \rightarrow \overline{m_2 n_2}$  and  $\overline{m_1 n_1} \rightarrow m_1 n_1$ , correspondingly. Besides, we may define that  $\hat{H}_{\dots}^{\uparrow\downarrow} \equiv \hat{H}_{\dots}^{\downarrow\uparrow}$ . As a result of a consistent perturbative study, we find that the correct zero-order MP states  $C_{\downarrow}|X_{\downarrow}\rangle + C_{\uparrow}|X_{\uparrow}\rangle$  and the correlation shifts are obtained from the equation

$$\mathcal{E} C_{\sigma} = \sum_{\sigma'} C_{\sigma'} M_{\sigma\sigma'}, \quad (4)$$

where the quantities  $M_{\sigma\sigma'}^{(1)} = \langle X_{\sigma} | \hat{\mathcal{H}}_{\text{int}} | X_{\sigma'} \rangle - E_0^{(1)} \delta_{\sigma, \sigma'}$  calculated within the first-order approximation vanish ( $E_0^{(1)}$  is the ground-state energy calculated to the first order); whereas the second-order approximation yields

solution of Eq. (4) is actually known. Indeed, the CP magnetoplasma mode in the zero order is written as  $\hat{K}_s|0\rangle \equiv \sqrt{N_{\phi}}(|X_{\downarrow}\rangle + \sqrt{2}|X_{\uparrow}\rangle)$ . Therefore, substituting  $C_{\downarrow}=1$ ,  $C_{\uparrow}=\sqrt{2}$ , and  $\mathcal{E}=\Delta_s \equiv 0$  into Eq. (4), we obtain two necessary identities:  $M_{\downarrow\uparrow} \equiv -M_{\downarrow\downarrow}/\sqrt{2}$  and  $M_{\uparrow\uparrow} \equiv M_{\downarrow\downarrow}/2$ .<sup>18</sup> Another root of the secular equation  $\det|M_{\sigma_1 \sigma_2} - \mathcal{E} \delta_{\sigma_1, \sigma_2}|=0$  is just the correlation shift for the AP mode and thus expressed in terms of the only matrix element (5):  $\mathcal{E}=\Delta_a=3M_{\downarrow\downarrow}/2$ . Second, considerable simplifications occur in the calculations associated with Eq. (5). It is evident that the  $\hat{H}_{\dots}^{\uparrow\downarrow}$  terms commuting with  $Q$  operators in Eq. (5) do not contribute to the result. However, due to Kohn's theorem, the  $\hat{H}_{\dots}^{\uparrow\downarrow}$  operators do not contribute either. Indeed, consider our ground state as a direct product of two fully polarized ground states:  $|0\rangle \equiv |0_{\downarrow}\rangle \otimes |0_{\uparrow}\rangle$ . Here  $|0_{\downarrow}\rangle$  is the  $\nu=1$  ground state with a positive  $g$  factor and  $|0_{\uparrow}\rangle$

is the  $\nu=2$  QH ferromagnet realized in the situation when the  $g$  factor is negative, but the Zeeman gap is larger than the cyclotron gap. In Eq. (5) all terms with the  $\hat{H}_{\dots}^{\uparrow\downarrow\dagger}$  operators act only on the  $\nu=1$  ground state and—taken together—yield zero because the sum of these terms would constitute the  $q=0$  correlation shift of Kohn's mode for the  $\nu=1$  QH ferromagnet.

Substituting the terms  $\hat{H}_{\dots}^{\uparrow\downarrow\dagger}$  and  $\hat{H}_{\dots}^{\downarrow\uparrow\dagger}$  into Eq. (5) and calculating the commutators according to commutation rules for exciton operators,<sup>14</sup> one finds

$$\Delta_a = -\frac{3m_e^*e^4}{2\kappa^2\hbar^2} \int_0^\infty q dq V(q)^2 G(q), \quad (6)$$

where

$$G(q) = \sum_{n_2=2}^{\infty} \left[ \frac{|h_{1n_2}|^2 (h_{00}^2 - 2h_{00}h_{11})}{n_2 - 1} + \frac{|h_{0n_2}|^2 (h_{00}^2 - 2h_{00}h_{11}) - |h_{01}h_{1n_2}|^2}{n_2} - \frac{|h_{01}h_{0n_2}|^2}{n_2 + 1} + \sum_{n_1=1}^{\infty} \left( \frac{|h_{1n_1}h_{1n_2}|^2}{n_1 + n_2 - 2} + \frac{|h_{1n_1}h_{0n_2}|^2 - |h_{0n_1}h_{1n_2}|^2}{n_1 + n_2 - 1} - \frac{|h_{0n_1}h_{0n_2}|^2}{n_1 + n_2} \right) \right]. \quad (7)$$

We emphasize that this result for  $\Delta_a$  includes all contributions to the second-order correction. In Eq. (7), terms containing only squared moduli of the  $h$  functions yield the direct Coulomb contribution. Terms containing  $\dots h_{00}h_{11}$  are of the exchange origin. (Thus the exchange contribution to the correlation shift is positive.)

In the strict 2D limit  $V(q)=1/q$ , and the correlation shift (6) and (7) is equal to  $-0.1044$  if expressed in the  $2Ry^* = m_e^*e^4/\kappa^2\hbar^2 \approx 11.34$  meV units. This value is nearly 2/3 of the correlation shift for the  $\nu=2$  cyclotron spin-flip mode

$\Delta_{SF} = -0.1534$ ,<sup>14</sup> which is in surprisingly good agreement with the experimental  $1/\nu$  dependence. Finally, substituting  $V(q)=F(q)/q$  into Eq. (6), one obtains a numerical result for the correlation shift of the zero-momentum AP mode at  $\nu=3$  (see Fig. 3). Here, the form factor  $F(q)$  is calculated with the usual self-consistent procedure.<sup>19</sup> The calculation result looks quite satisfactory compared to the ILS data, if one takes into account that under specific experimental conditions the quantity  $r_c$  can only be considered as a “small parameter” with great reserve.

To conclude, we outline the general meaning of the presented results. It is known that optical methods (including ILS), being in practice the only tool for *direct study* of cooperative excitations in a correlated 2DEG, suffer from an inevitable disadvantage: small momenta of studied excitations are far off the interesting region, corresponding to inverse values of mean electron-electron distance. Besides, studying the symmetric MP spectra, one only comes to the results well described by the classical plasma formula (1), which can be rewritten as  $E_s/\hbar \approx \omega_c + \Omega_p^2/2\omega_c$  ( $\Omega_p$  to denote the 2D plasma frequency). Therefore the CP magnetoplasma modes are actually classical plasma oscillations irrelevant to any quantum effects. Contrary to this, homogeneous but antisymmetric modes, namely, the AP mode in a QH ferromagnet and the cyclotron spin-flip mode in an unpolarized QH system are *quantum excitations* even at zero  $q$ —related to the existence of both the spin-up and spin-down subsystems. The correlation shift, measured in effective rydbergs, represents therefore a purely quantum effect. In particular, it includes exchange corrections, which can be taken into account neither by classical plasma calculations nor by the random-phase approximation (RPA) approach. Quantum origin, common for both types of antisymmetric excitation, seems to be a reason why both second-order correlation shifts are empirically well described by the same  $1/\nu$  dependence shown in Fig. 3.

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<sup>11</sup>Positive value of  $\Delta_a$  would not remove the degeneracy but only shifts the degeneracy point to a nonzero value of  $q \sim r_c/l_B$ . Such a feature would be physically unjustified.

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<sup>18</sup>Calculation of all elements  $M_{\sigma_1\sigma_2}$  can be performed by means of formula (5) and by similar ones. This calculation, giving values satisfying the necessary identities, serves as a good check for the correctness of our theory.

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